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A STUDY OF SOME METHODS FOR MEASURING CKM CP VIOLATING PHASES

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Abstract

We study the influence of penguin (especially, electroweak penguin) effects on some methods of measuring the angles α, β, and γ in the CKM unitarity triangle. We use next-to-leading order effective Hamiltonian, and present numerical estimates based on the factorization approximation. We find that some techniques suggested in the literature, especially for α determination, are not workable in light of the electroweak penguin effects. Nevertheless, there are methods that would work for each angle determination. For angle β we consider $B \to D^+D^-$ mode and estimate the penguin contamination. For angle γ we consider a method based on SU(3) symmetry and carefully consider SU(3) breaking effects. We point out regions in the parameter space where this method could be used reliably.

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I. INTRODUCTION

The measurement of the ϵ-parameter in the $K^0 - \bar{K}^0$ meson system is the only direct evidence for CP violation in the laboratory [1]. Many models have been proposed to explain this phenomena [2,3]. The Standard Model (SM) of three generations with the source for CP violation arising from the phases in the Cabibbo-Kobayashi-Maskawa (CKM) matrix is consistent with the experiment [2]. It is necessary to perform more experiments to find out the source or sources of CP violation and to test the CKM model. A unique feature of the CKM model for CP violation is that the CKM matrix is a 3×3 unitary matrix. Due to the unitarity property, when summed over the row or column of matrix elements V_{ij} times complex conjugate matrix elements V^*_{ik}, the following equation holds,

$$\sum_i V_{ij} V^*_{ik} = \delta_{jk} \ . \tag{1}$$

This equation defines a triangle when $j \neq k$. For example, for $j = d$ and $k = b$ a triangle shown in Fig. 1 with three angles $\alpha \equiv \mathrm{Arg}(-V_{td}V^*_{tb}/V^*_{ub}V_{ud})$, $\beta \equiv \mathrm{Arg}(-V_{cd}V^*_{cb}/V^*_{tb}V_{td})$, and $\gamma \equiv \mathrm{Arg}(-V_{ud}V^*_{ub}/V^*_{cb}V_{cd})$ is defined. These angles are related to phases of the CKM matrix elements. In Wolfenstein parametrization the three angles of the triangle are given by $\alpha = \mathrm{Arg}\ (-V_{td}/V^*_{ub})$, $\beta = \mathrm{Arg}\ V^*_{td}$, $\gamma = \mathrm{Arg}\ V^*_{ub}$ [4]. The sum of these three angles must be equal to 180^0. This is a unique feature of the CKM model for CP violation. This property provides an important way to check the validity of the CKM model if enough independent measurements of the sides and angles of the triangle can be performed experimentally. B meson decays provide a fertile ground to carry out such a test [5].

Many methods have been suggested for measuring α, β and γ using B decays [5–10]. One class of methods involve the measurements of CP asymmetries in time evolution of B^0 decays into CP eigenstates. Such methods make it possible to measure the three angles of the unitarity triangle independently and without hadronic uncertainties if amplitudes depending on a single CKM phase dominate the decay process. For instance [5], $\sin 2\alpha$, $\sin 2\beta$, and $\sin 2\gamma$ can be measured in decays $B^0 \to \pi^+\pi^-$, $B^0 \to \Psi K^0_S$, and $B^0_s \to \rho K^0_S$, respectively.

However, in practice due to simultaneous contributions to these decays from the tree and loop (penguin) effects, the measurements are much more difficult. In many cases the CKM phases can not be extracted just from these asymmetries. Additional information is needed. Some methods have been developed to extract CKM phases using relations based on isospin or flavor SU(3) symmetries to isolate the CKM phase of one type of amplitude such that the CKM phase determined is not contaminated by the presence of other amplitudes [7–16]. Some of these relations when used for charged B meson decay modes, also provide new methods to measure some of the CKM phase angles.

There are two types of penguin contributions, the strong and the electroweak penguins. Naively, one would expect that the electroweak penguin effects are suppressed by a factor of α_{em}/α_s compared with the strong penguin effects, and therefore can be negelected. Many previous methods for measuring the CKM phases have explicitly made such assumption. In a recent paper by two of us [12], we pointed out that this assumption turns out to be wrong for large top quark mass. Some of the methods proposed in the literature become invalid when the electroweak penguin effects are included. In this paper we study the effects of the electroweak penguin on several methods for measuring the CKM phases in the literature. We will concentrate on methods based on B decays. There are methods based on B_s decays, which are much more difficult to perform experimentally, and we will not discuss them in this paper.

The paper is organized as following: In section II we present the full effective Hamiltonian responsible for B decays, and some isospin and SU(3) analysis of the decay amplitudes; In section III, IV and V we study the influence of penguin effects on some methods for measuring the CKM phases α, β and γ, respectively; And in section VI we present our conclusions.

II. THE EFFECTIVE HAMILTONIAN FOR B MESON DECAYS

The effective Hamiltonian up to one loop level in electroweak interaction for hadronic B decays can be written as

$$H_{\Delta B=1} = \frac{4G_F}{\sqrt{2}}[V_{ub}V^*_{uq}(c_1O^u_1 + c_2O^u_2) + V_{cb}V^*_{cq}(c_1O^c_1 + c_2O^c_2) - V_{tb}V^*_{tq}\sum_{i=3}^{12} c_iO_i] + H.C., \quad (2)$$

where O_i's are defined as

$$\begin{aligned}
&O^f_1 = \bar{q}_\alpha\gamma_\mu L f_\beta \bar{f}_\beta\gamma^\mu L b_\alpha, \quad O^f_2 = \bar{q}\gamma_\mu L f \bar{f}\gamma^\mu L b, \\
&O_{3(5)} = \bar{q}\gamma_\mu L b \Sigma \bar{q}'\gamma^\mu L(R)q', \quad O_{4(6)} = \bar{q}_\alpha\gamma_\mu L b_\beta \Sigma \bar{q}'_\beta\gamma^\mu L(R)q'_\alpha, \\
&O_{7(9)} = \frac{3}{2}\bar{q}\gamma_\mu L b \Sigma e_{q'}\bar{q}'\gamma^\mu R(L)q', \quad O_{8(10)} = \frac{3}{2}\bar{q}_\alpha\gamma_\mu L b_\beta \Sigma e_{q'}\bar{q}'_\beta\gamma^\mu R(L)q'_\alpha, \\
&O_{11} = \frac{g_s}{32\pi^2}m_b\bar{q}\sigma_{\mu\nu}RT_a b G^{\mu\nu}_a \ , \quad Q_{12} = \frac{e}{32\pi^2}m_b\bar{q}\sigma_{\mu\nu}RbF^{\mu\nu} \ , \qquad (3)
\end{aligned}$$

where $L(R) = (1 \mp \gamma_5)/2$, f can be u or c quark, q can be d or s quark, and q' is summed over u, d, s, and c quarks. α and β are the color indices. T^a is the SU(3) generator with the normalization $Tr(T^aT^b) = \delta^{ab}/2$. $G^{\mu\nu}_a$ and $F_{\mu\nu}$ are the gluon and photon field strength, respectively. c_i are the Wilson Coefficients (WC). O_1, O_2 are the tree level and QCD corrected operators. O_{3-6} are the gluon induced strong penguin operators. O_{7-10} are the electroweak penguin operators due to γ and Z exchange, and "box" diagrams at loop level. The operators $O_{11,12}$ are the dipole penguin operators.

The WC's c_i at a particular scale μ are obtained by first calculating the WC's at m_W scale and then using the renormalization group equation to evolve them to μ. We have carryed out this analysis using the next-to-leading order QCD corrected WC's following Ref. [17]. Using $\alpha_s(m_Z) = 0.118$, $\alpha_{em}(m_Z) = 1/128$, $m_t = 176$ GeV and $\mu \approx m_b = 5$ GeV, we obtain from top-quark contribution [18]

$$\begin{aligned}
&c_1 = -0.3125, \quad c_2 = 1.1502, \quad c_3 = 0.0174, \quad c_4 = -0.0373, \\
&c_5 = 0.0104, \quad c_6 = -0.0459, \quad c_7 = -1.050 \times 10^{-5}, \\
&c_8 = 3.839 \times 10^{-4}, \quad c_9 = -0.0101, \quad c_{10} = 1.959 \times 10^{-3}. \qquad (4)
\end{aligned}$$

It is interesting to note that the coefficient c_9 arising from electroweak penguin comtribution is not much smaller than coefficients of the strong penguin. This enhancement is caused by a term in the electroweak penguin contributions in which the WC is proportional to the

square of the top quark mass due to Z exchange. In some application we will need absorptive parts of c and u loop contributions. These are given in Ref. [18].

The coefficients for the dipole penguin operators at the two loop level have the following values [19]:

$$c_{11} = -0.299 \ , \quad c_{12} = -0.634 \ . \tag{5}$$

To study exclusive B decays, we also need to transform the quark operators into hadrons. This is a difficult task. At present there is no reliable way to carry out this calculation. Nevertheless, many models and suggestions have been made to provide some handle on the related hadronic matrix elements. Symmetry considerations provide very powerful constraints on the matrix elements and relate different decay amplitudes. Isospin and flavor SU(3) symmetry are two very useful symmetries used in the analysis in this paper.

We can always parametrize the decay amplitude of B that arises from quark subprocess $b \to u\bar{u}q$ as

$$\bar{A} =< final\ state|H^q_{eff}|B >= V_{ub}V^*_{uq}T(q) + V_{tb}V^*_{tq}P(q) \ , \tag{6}$$

where $T(q)$ contains the *tree* as well as *penguin* contributions, while $P(q)$ contains purely *penguin* contributions.

When q is fixed, isospin symmetry relates some of the decay amplitudes generated by the effective Hamiltonian. In the case for $q = d$, isospin symmetry relates decay amplitudes for different $B \to \pi\pi$ or $B \to \rho\pi$ decays. It also gives information on which operators in the effective Homiltonian contribute to certain isospin decay amplitudes. This is very important for our discussions in the rest of the paper.

For $q = d$, the tree operators $O_{1,2}$ and the electroweak penguin operators O_{7-10} contain $\Delta I = 1/2$ and $\Delta I = 3/2$ interactions whereas the strong operators O_{3-6} and the dipole penguin operators $O_{11,12}$ contain only $\Delta I = 1/2$ interaction. In the case of $B \to \pi\pi$, Bose symmetry requires $\pi\pi$ to be in $I = 0$ or $I = 2$ state. Since B meson is a $I = 1/2$ state, we immediately know that the strong and dipole penguin operators will not contribute to $I = 2$

decay amplitude, for example $B^- \to \pi^-\pi^0$. In the case for $B \to \rho\pi$, the final states can have $I = 0$, 1 or 2 states. We also easily see that the strong and dipole penguin operators will not contribute to the $I = 2$ decay amplitude.

For $q = s$. The tree operators $O_{1,2}$ and the electroweak penguin operators O_{7-10} contain $\Delta I = 0$ and 1, and the strong and dipole penguin operators O_{3-6} and $O_{11,12}$ contain $\Delta I = 0$ interaction only. In $B \to \pi K$ decays, the combinations, $\bar{A}(B^- \to \pi^0 K^-) + \bar{A}(\bar{B}^0 \to \pi^0 \bar{K}^0)$ and $\bar{A}(\bar{B}^0 \to \pi^+ K^-) - \bar{A}(B^- \to \pi^- \bar{K}^0)$ contain only $I = 3/2$. We immediately know that the strong and dipole penguin will not contribute to these amplitudes.

Isospin symmetry will not relate the amplitudes between the amplitudes with $q = d$ and amplitudes with $q = s$. However, if one enlarges the symmetry group to the flavor SU(3), these can be related. The isospin relations will still be maintaned because isospin is a subgroup of flavor SU(3) symmetry. We shall now use SU(3) symmetry to obtain some relations which will be used in our later discussions.

SU(3) relations for B decays have been studied by several authors [13,14,20,21]. The operators $O_{1,2}$, $O_{3-6,11,12}$, and O_{7-10} transform under SU(3) symmetry as $\bar{3}_a + \bar{3}_b + 6 + \overline{15}$, $\bar{3}$, and $\bar{3}_a + \bar{3}_b + 6 + \overline{15}$, respectively. In general, we can write the SU(3) invariant amplitude for B decay to two octet pseudoscalar mesons. For the T amplitude, for example, we have

$$\begin{aligned}
T = &A^T_{(\bar{3})} B_i H(\bar{3})^i (M^k_l M^l_k) + C^T_{(\bar{3})} B_i M^i_k M^k_j H(\bar{3})^j \\
&+ A^T_{(6)} B_i H(6)^{ij}_k M^l_j M^k_l + C^T_{(6)} B_i M^i_j H(6)^{jk}_l M^l_k \\
&+ A^T_{(\overline{15})} B_i H(\overline{15})^{ij}_k M^l_j M^k_l + C^T_{(\overline{15})} B_i M^i_j H(\overline{15})^{jk}_l M^l_k \,, \qquad (7)
\end{aligned}$$

where $B_i = (B^-, \bar{B}^0, \bar{B}^0_s)$ is a SU(3) triplet, M^j_i is the SU(3) pseudoscalar octet, and the matrices H represent the transformation properties of the operators O_{1-12}. $H(6)$ is a traceless tensor that is antisymmetric on its upper indices, and $H(\overline{15})$ is also a traceless tensor but is symmetric on its upper indices. We can easily see that the strong and dipole penguin operators only contribute to A_3 and C_3.

For $q = d$, the non-zero entries of the H matrices are given by

$$H(\bar{3})^2 = 1 \,, \quad H(6)^{12}_1 = H(6)^{23}_3 = 1 \,, \quad H(6)^{21}_1 = H(6)^{32}_3 = -1 \,,$$

$$H(\overline{15})^{12}_1 = H(\overline{15})^{21}_1 = 3\ ,\ H(\overline{15})^{22}_2 = -2\ ,\ H(\overline{15})^{32}_3 = H(\overline{15})^{23}_3 = -1\ . \tag{8}$$

For $q = s$, the non-zero entries are

$$\begin{aligned} H(\bar{3})^3 = 1\ ,\ \ H(6)^{13}_1 = H(6)^{32}_2 = 1\ ,\ \ H(6)^{31}_1 = H(6)^{23}_2 = -1\ , \\ H(\overline{15})^{13}_1 = H(\overline{15})^{31}_1 = 3\ ,\ H(\overline{15})^{33}_3 = -2\ ,\ H(\overline{15})^{32}_2 = H(\overline{15})^{23}_2 = -1\ . \end{aligned} \tag{9}$$

In terms of the SU(3) invariant amplitudes, the decay amplitudes $T(\pi\pi)$, $T(\pi K)$ for $\bar{B}^0 \to \pi\pi$, $\bar{B}^0 \to \pi K$ are given by

$$\begin{aligned} T(\pi^+\pi^-) &= 2A^T_{(\bar{3})} + C^T_{(\bar{3})} - A^T_{(6)} + C^T_{(6)} + A^T_{(\overline{15})} + 3C^T_{(\overline{15})}\ , \\ T(\pi^0\pi^0) &= \frac{1}{\sqrt{2}}(2A^T_{(\bar{3})} + C^T_{(\bar{3})} - A^T_{(6)} + C^T_{(6)} + A^T_{(\overline{15})} - 5C^T_{(\overline{15})})\ , \\ T(\pi^-\pi^0) &= \frac{8}{\sqrt{2}}C^T_{(\overline{15})}\ , \\ T(\pi^-\bar{K}^0) &= C^T_{(\bar{3})} + A^T_{(6)} - C^T_{(6)} + 3A^T_{(\overline{15})} - C^T_{(\overline{15})}\ , \\ T(\pi^0K^-) &= \frac{1}{\sqrt{2}}(C^T_{(\bar{3})} + A^T_{(6)} - C^T_{(6)} + 3A^T_{(\overline{15})} + 7C^T_{(\overline{15})})\ , \\ T(\pi^+K^-) &= C^T_{(\bar{3})} - A^T_{(6)} + C^T_{(6)} - A^T_{(\overline{15})} + 3C^T_{(\overline{15})}\ , \\ T(\pi^0\bar{K}^0) &= -\frac{1}{\sqrt{2}}(C^T_{(\bar{3})} - A^T_{(6)} + C^T_{(6)} - A^T_{(\overline{15})} - 5C^T_{(\overline{15})})\ , \\ T(\eta_8K^-) &= \frac{1}{\sqrt{6}}(-C^T_{(\bar{3})} - A^T_{(6)} + C^T_{(6)} - 3A^T_{(\overline{15})} + 9C^T_{(\overline{15})})\ , \end{aligned} \tag{10}$$

We also have similar relations for the amplitude $P(q)$. The corresponding SU(3) invariant amplitudes will be denoted by A^P_i and C^P_i. It is easy to obtain the following relations from above:

$$\begin{aligned} \sqrt{2}\bar{A}(\bar{B}^0 \to \pi^0\pi^0) + \sqrt{2}\bar{A}(B^- \to \pi^-\pi^0) &= \bar{A}(\bar{B}^0 \to \pi^+\pi^-)\ , \\ \bar{A}(\bar{B}^0 \to \pi^+K^-) + \bar{A}(B^- \to \pi^-\bar{K}^0) + \sqrt{2}\bar{A}(\bar{B}^0 \to \pi^0\bar{K}^0) &= \sqrt{2}\bar{A}(B^- \to \pi^0K^-)\ , \\ \sqrt{2}\bar{A}(B^- \to \pi^0K^-) - 2\bar{A}(B^- \to \pi^-\bar{K}^0) &= \sqrt{6}\bar{A}(B^- \to \eta_8K^-)\ . \end{aligned} \tag{11}$$

One expects the hadronic matrix elements arising from quark operators to be the same order of magnitudes, the relative strength of the amplitudes T and P are predominantly determined by their corresponding WC's in the effective Hamiltonian. However, in order

to numerically compare contributions from different operators, we have to rely on model calculations. In our later analysis, when such numerical calculations are required, we will use factorization approximation. These numerical numbers may not be accurate, but they will serve well in providing an idea of the validity of certain assumptions made.

III. MEASUREMENT OF THE PHASE ANGLE α

In this section we study the electroweak penguin effects on several methods proposed for measuring the CKM phase angle α.

(1) From time dependent asymmetries in $B \to \pi\pi$ decays

Let us first consider the standard method to measure the CKM phase α in $B \to \pi\pi$ [5,7]. We present it here to set up our notations and also to clarify some issues. The time-dependent rate for initially pure B^0 or $\bar{B}^0$ states to decay into a final CP eigenstate, for example $\pi^+\pi^-$ at time t is [5]

$$\Gamma(B^0(t) \to \pi^+\pi^-) = |A|^2 e^{-\Gamma t}[\frac{1+|\lambda|^2}{2} + \frac{1-|\lambda|^2}{2}\cos(\Delta M t) - Im\lambda \sin(\Delta M t)],$$
$$\Gamma(\bar{B}^0(t) \to \pi^+\pi^-) = |A|^2 e^{-\Gamma t}[\frac{1+|\lambda|^2}{2} - \frac{1-|\lambda|^2}{2}\cos(\Delta M t) + Im\lambda \sin(\Delta M t)], \tag{12}$$

where λ is defined as

$$\lambda \equiv \frac{q}{p}\frac{\bar{A}}{A} \tag{13}$$

with $A \equiv A(B^0 \to \pi^+\pi^-)$ and $\bar{A} \equiv \bar{A}(\bar{B}^0 \to \pi^+\pi^-)$. Here p and q are given by the relations

$$|B_{L,H} >= p|B^0 > \pm q|\bar{B}^0 > \tag{14}$$

for the two mass eigenstates B_H and B_L. In the SM,

$$\frac{q}{p} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\ . \tag{15}$$

The parameter $Im\lambda$ can be determined by mesuring the coefficient in CP asymmetry in time evolution varying with time as a sine function.

Using the effective Hamiltonian given in section II, we can parametrize the decay amplitude in general as

$$\bar{A} = V_{ub}V_{ud}^{*}T + V_{tb}V_{td}^{*}P \ . \tag{16}$$

If the penguin amplitude can be neglected, we have

$$Im\lambda = Im(\frac{V_{tb}^{*}V_{td}}{V_{tb}V_{td}^{*}}\frac{V_{ub}V_{ud}^{*}}{V_{ub}^{*}V_{ud}}) = \sin(2\alpha) \ . \tag{17}$$

The phase α can therefore be determined. Similarly one can obtain α from $\bar{B}^0 \to \pi^0\pi^0$.

When the pegnuin effects are included, $Im\lambda$ is not equal to $\sin(2\alpha)$ any more. One finds [12]

$$\Delta\sin(2\alpha) \equiv Im\lambda - \sin(2\alpha) = -R\frac{2\cos\delta\sin\alpha + \sin(2\alpha)(R - 2\cos(\delta+\alpha))}{1 + R^2 - 2R\cos(\delta+\alpha)} \ , \tag{18}$$

where $R = |P/T|$, and δ is the relative strong rescattering phase between the T and P amplitudes. It was estimated in Ref. [12] using factorization approximation that $R = 7\%$. Using this number it was found that the error on the determination of the phase α can be as large as 12^0. The error is even larger if $\bar{B}^0 \to \pi^0\pi^0$ is used, where R is estimated to be 23%.

(2) Gronau-London method

When penguin effects are included, the parameter $Im\lambda$ can be parametrized as [7]

$$Im\lambda = \frac{|\bar{A}|}{|A|}\sin(2\alpha + \theta) \ . \tag{19}$$

The ratio $|\bar{A}|/|A|$ can be determined by measuring the coefficient of the term in CP asymmetry in time evolution varying as cosine function at asymmetric colliders, and also at symmetric colliders [22]. If θ can be determined independently, the phase α can also be determined. To determine θ, Gronau and London [7] proposed using isospin relation

$$\sqrt{2}\bar{A}(\bar{B}^0 \to \pi^0\pi^0) + \sqrt{2}\bar{A}(B^- \to \pi^-\pi^0) = \bar{A}(\bar{B}^0 \to \pi^+\pi^-) \ , \tag{20}$$

obtained in eq.(11), and similar relation for the CP-conjugate amplitudes for the corresponding anti-particle decays. If all the six amplitudes can be measured, the angle θ can

be determined up to two fold ambiguity as shown in Fig.2. In this method GL argued, because $\bar{B}^- \to \pi^-\pi^0$ has only $I = 2$ amplitude in the final state to which only $\Delta I = 3/2$ interaction in the Hamiltonian contribute, that $\bar{A}(B^- \to \pi^-\pi^0)$ and $\bar{A}(B^+ \to \pi^+\pi^0)$ have no contribution from penguin operators and therefore $|\bar{A}(B^- \to \pi^-\pi^0)| = |A(B^+ \to \pi^+\pi^0)|$. The strong penguin only has $\Delta I = 1/2$ interaction, so the strong penguin does not contribute to this decay. If the electroweak penguin effects are neglected, the equlity $|\bar{A}(B^- \to \pi^-\pi^0)| = |A(B^+ \to \pi^+\pi^0)|$ is exact. The electroweak penguin actually contains $\Delta I = 3/2$ interaction, and contributes to the decay amplitude. However, since the electroweak penguin $<$ strong penguin $<$ tree contribution for this process, the contribution is expected to be very small. An estimate based on factorization gives less than 3% [12]. The inclusion of the electroweak penguin effects can be safely neglected. This method, in principle, can determine the phase α at a few percent level.

(3) Hamzaoui-Xing method

A method to measure the phase α without using CP asymmetry in time evolution has also been proposed recently by Hamzaoui and Xing [11]. We show that this method actually fails. Based on isospin consideration and factorization approximation, they parametrized the decay amplitudes for $B \to \pi\pi$ as follows:

$$\begin{aligned}
A_{+0} &\equiv A(B^+ \to \pi^+\pi^0) = -\frac{1+a}{\sqrt{2}} T e^{i\gamma}, \\
A_{+-} &\equiv A(B^0 \to \pi^+\pi^-) = -T e^{i\gamma} - P_{+-} e^{i(\delta-\beta)}, \\
A_{00} &\equiv A(B^0 \to \pi^0\pi^0) = -\frac{a}{\sqrt{2}} T e^{i\gamma} + \frac{1}{\sqrt{2}} P_{00} e^{i(\delta-\beta)},
\end{aligned} \tag{21}$$

where the T and P_{ij}'s are the tree and penguin amplitudes, respectively. δ denotes the strong relative phase between the penguin and tree amplitudes. The parameter a denotes the color-mismatched suppressed contribution in the tree amplitudes. If electroweak penguin effects are neglected,

$$P_{+-} = P_{00} \equiv P. \tag{22}$$

One then obtains

$$\cos(\alpha+\delta) = \frac{1}{2\chi}[1+\chi^2-(1+a)^2 R_{+-}],$$
$$\cos(\alpha-\delta) = \frac{1}{2\chi}[1+\chi^2-(1+a)^2 \bar{R}_{+-}], \tag{23}$$

where $\chi = P/T$, and R_{+-} and $\bar{R}_{+-}$ are the measurable quantities defined as $R_{ij} = |A_{ij}|^2/(|A_{+0}|^2+|\bar{A}_{-0}|^2)$ ($\bar{A}$ denotes the CP-conjugate amplitude of A). The parameters a and χ can also be expressed in terms of experimental measurables,

$$a = -2\frac{\bar{R}_{00}-R_{00}}{\bar{R}_{+-}-R_{+-}} ,$$
$$\chi = \sqrt{(1+a)(aR_{+-}+2R_{00})-a} . \tag{24}$$

Therefore, if the assumptions are correct, the phase α could be determined.

In order to obtain the above equations, a cruical assumption has been made that the parameter $\chi_{+-} = P_{+-}/T$ is equal to $\chi_{00} = P_{00}/T$. This equality is true only if there is no electroweak penguin contribution. The validity of the proposed method can be checked when electroweak penguin effects are included. Using factorization approximation, we obtain

$$\begin{aligned}
T &= -\frac{G_F}{\sqrt{2}}|V_{ub}V_{ud}|(\xi c_1+c_2)T_{\pi\pi} ,\\
P_{+-} &= \frac{G_F}{\sqrt{2}}|V_{ub}V_{ud}|[\xi c_3+c_4+2(\xi c_5+c_6+\xi c_7+c_8)X_1+\xi c_9+c_{10}]T_{\pi\pi} ,\\
P_{00} &= \frac{G_F}{\sqrt{2}}|V_{ub}V_{ud}|[\xi c_3+c_4+2(\xi c_5+c_6\frac{1}{2}\xi c_7-\frac{1}{2}c_8)X_2+\frac{3}{2}(c_7+\xi c_8-c_9-\xi c_{10})\\
&\quad -\frac{1}{2}(\xi c_9+c_{10})]T_{\pi\pi} ,
\end{aligned} \tag{25}$$

where $\xi = 1/N$ with N being the number of color, and

$$X_1 = \frac{m_\pi^2}{(m_b-m_u)(m_d+m_u)} ,\quad X_2 = \frac{m_\pi^2}{2m_d(m_b-m_d)} ,$$
$$T_{\pi\pi} = if_\pi[f^+_{B\pi}(m_\pi^2)(m_B^2-m_\pi^2)+f^-_{B\pi}(m_\pi^2)m_\pi^2] . \tag{26}$$

In our calculations we will use $f_\pi = 132$ MeV, and the form factors $f^{\pm}_{B\pi}$ in Ref. [23]. For b quark mass, we use $m_b = 5$ GeV, and for the light quark masses $m_u = 5.6$ MeV, $m_d = 8.7$ MeV. We also treat ξ as a free parameter and use the experimentally favoured value $\xi \approx 1/2$ [24]. We find that for $B^0 \to \pi^0\pi^0$ the ratio of the electroweak penguin to the strong penguin

amplitude is quite large (about 33%), while that ratio for $B^0 \to \pi^+\pi^-$ is only 5.7%. In $B^0 \to \pi^0\pi^0$ decay, the electroweak penguin contributions have the opposite sign to the strong penguin contributions and reduce the total penguin effects. We find the values for χ_{+-} and χ_{00} are very different, $\chi_{+-}/\chi_{00} = 1.71$. Here we have neglected small contributions from the dipole penguin operators which do not affect the result significantly. It is clear that the assumption $\chi_{+-} = \chi_{00}$ is badly violated, and therefore the method proposed in Ref. [11] fails completely.

(4) Using $B \to \rho\pi$ decays

In Ref. [8,9] it was pointed out that neutral $B \to \rho\pi$ can be used to extract α without ambiguities due to penguin contributions. It was shown in Ref. [8,9] that by studying full Dalitz plot and time dependence for $B^0 \to \pi^+\pi^-\pi^0$, the amplitudes and phases of $B^0 \to \rho\pi$ decays, $S_3 = A(B^0 \to \rho^+\pi^-)$, $S_4 = A(B^0 \to \rho^-\pi^+)$, $S_5 = -2A(B^0 \to \rho^0\pi^0)$, and their CP-conjugate amplitudes can be determined. Isospin analysis then shows that the sum $S = S_3 + S_4 + S_5$ has only $I = 2$ amplitude. Therefore it arises from $\Delta I = 3/2$ interaction. If electroweak penguin effects are neglected, S has only tree contribution which contains the phase angle γ. Combined with angle β from the mixing parameter q/p in B^0-$\bar{B}^0$, and considering coefficient of $\sin(\Delta M t)$ (see eq.(12)), the phase α can be determined.

Since the electroweak penguin contains $\Delta I = 3/2$ interation, when its effects are included, S contains in addition to the tree amplitude S_{tree}, also the electroweak penguin amplitude S_{ew} which has a different weak phase. The determination of the phase α will therefore be contaminated. In the factorization approximation, we obtain

$$
\begin{aligned}
S_{tree} &= \frac{G_F}{\sqrt{2}} V_{ub}^* V_{ud}(1+\xi)(c_1+c_2)(\tilde{C}+\tilde{T}), \\
S_{ew} &= \frac{G_F}{\sqrt{2}} V_{tb}^* V_{td}[3(\xi c_7 + c_8) X \tilde{T} - \frac{3}{2}(c_7 + \xi c_8)(\tilde{C} - \tilde{T}) \\
&\quad - \frac{3}{2}(1+\xi)(c_9 + c_{10})(\tilde{C}+\tilde{T})],
\end{aligned}
\tag{27}
$$

where

$$
X = \frac{m_\pi^2}{2m_d(m_b+m_d)},
$$

$$\tilde{C} = 2m_\rho(\epsilon_\rho^* \cdot p_\pi) f_\rho f_{B\pi}^+(m_\rho^2),$$
$$\tilde{T} = -2m_\rho(\epsilon_\rho^* \cdot p_\pi) f_\pi A_0^{B\rho}(m_\pi^2). \tag{28}$$

Here we have neglected a small contribution due to annihilation effects. Note that the strong and dipole penguin operators do not contribute to S. For our numerical calculations we will use $f_\rho = 221$ MeV, and the form factors $f_{B\pi}^+$ and $A_0^{B\rho}$ calculated in Ref. [23]. We find

$$\frac{|S_{ew}|}{|S_{tree}|} \approx 1.4\%(|V_{td}|/|V_{ub}|). \tag{29}$$

We see that the ratio of the electroweak penguin to the tree contribution is very small and S is dominated by the tree contribution. Therefore, this method of measuring the phase α is good to a few percent level.

(5) Using $B \to \pi K$ decays

A method for measuring the phase angle α has also been proposed using $|\Delta S| = 1$ B decay processes by Nir and Quinn [9]. Measurement of CP asymmetry in time evolution in $B^0 \to \pi^0 K_S$ will be able to determine the parameter

$$Im\lambda = Im(\frac{q}{p}\frac{\bar{A}(\bar{B}^0 \to \pi^0 K_S)}{A(B^0 \to \pi^0 K_S)}) = Im(e^{-2i(\beta+\gamma)}\frac{e^{2i\gamma}\bar{A}(\bar{B}^0 \to \pi^0 K_S)}{A(B^0 \to \pi^0 K_S)});. \tag{30}$$

If penguin effects are neglected, $\bar{A}/A = e^{-2i\gamma}$ and so $Im\lambda = \sin(2\alpha)$. For this decay it is obviously wrong to neglect the penguin effects because the penguin contribuitons are enhanced by a factor of $|V_{tb}V_{ts}^*/V_{ub}V_{us}^*| \approx 50$ [26] compared to the tree contributions. Even though the WC's of the penguin operators are smaller than the tree WC's, the net penguin contributions may be larger than the tree contributions. In Ref. [9] a method have been proposed to overcome difficulties associated with strong penguin effects by determining $e^{2i\gamma}\bar{A}(\bar{B}^0 \to \pi^0 K_S)/A(B^0 \to \pi^0 K_S)$ directly from isospin analysis. This method requires measurement of the decay amplitudes, $A(\bar{B}^0 \to \pi^0 \bar{K}^0)$, $\bar{A}(\bar{B}^0 \to \pi^+ \bar{K}^-)$, $\bar{A}(B^- \to \pi^0 \bar{K}^-)$, $\bar{A}(B^- \to \pi^- \bar{K}^0)$, and their CP-conjugatey amplitudes. The strong and dipole penguin operators do not contribute to the follwoing combinations because they are $I = 3/2$ amplitudes, which can also be easily seen from eq.(10),

$$
\begin{aligned}
\bar{U} &\equiv \frac{1}{2}\bar{A}(B^- \to \pi^0 K^-) + \frac{1}{2}\bar{A}(\bar{B}^0 \to \pi^0 \bar{K}^0) \ , \\
\bar{V} &\equiv -\frac{1}{2}\bar{A}(B^- \to \pi^- \bar{K}^0) + \frac{1}{2}\bar{A}(\bar{B}^0 \to \pi^+ K^-) \ , \\
U &\equiv \frac{1}{2}A(B^+ \to \pi^0 K^+) + \frac{1}{2}A(B^0 \to \pi^0 K^0) \ , \\
V &\equiv -\frac{1}{2}A(B^+ \to \pi^+ K^0) + \frac{1}{2}A(B^0 \to \pi^- K^+) \ .
\end{aligned} \tag{31}
$$

If the electroweak penguin effects are neglected,

$$
U = \bar{U}e^{i2\gamma} \ , \ \ V = \bar{V}e^{i2\gamma} \ . \tag{32}
$$

When the eight decay amplitudes for $B \to \pi K$ are measured, using the conditions in eq.(32), the quadrilaterals in eq.(11)

$$
\begin{aligned}
\tilde{A}(B^- \to \pi^0 K^-) - \frac{1}{\sqrt{2}}\tilde{A}(B^- \to \pi^- \bar{K}^0) &= \tilde{A}(\bar{B}^0 \to \pi^0 \bar{K}^0) + \frac{1}{\sqrt{2}}\tilde{A}(\bar{B}^0 \to \pi^+ K^-) \ , \\
A(B^+ \to \pi^0 K^+) - \frac{1}{\sqrt{2}}A(B^+ \to \pi^+ K^0) &= A(B^0 \to \pi^0 K^0) + \frac{1}{\sqrt{2}}A(B^0 \to \pi^- K^+) \ ,
\end{aligned} \tag{33}
$$

can be constructed with $U + V$ being a common diagonal. Here $\tilde{A}$'s are defined as $\tilde{A}(B \to \pi K) \equiv \bar{A}(B \to \pi K)e^{2i\gamma}$. Once these quadrilaterals are constructed, the quantity $e^{i2\gamma}\bar{A}(\bar{B}^0 \to \pi^o K_S)/A(B^0 \to \pi^0 K_S)$ can be easily determined. Combining this information with eq.(30), the phase α can be determined.

However, since the electroweak penguin operators also contain $\Delta I = 1$ interaction, we have to check the validity of the relations of eq.(32). In factorization approximation we find the magnitudes of the amplitudes U, V, $\bar{U}$, and $\bar{V}$:

$$
\begin{aligned}
U = \frac{G_F}{\sqrt{2}}\frac{1}{2\sqrt{2}} \ \{ \ & V_{ub}^* V_{us}[2(c_1 + \xi c_2)C' + (\xi c_1 + c_2)(T' + A')] \\
& - V_{tb}^* V_{ts}[-3(c_7 + \xi c_8 - c_9 - \xi c_{10})C' + 3(\xi c_7 + c_8)(YT' + ZA') \\
& + \frac{3}{2}(\xi c_9 + c_{10})(T' + A')]\}, \\
V = \frac{G_F}{\sqrt{2}}\frac{1}{2\sqrt{2}} \ \{ \ & V_{ub}^* V_{us}[(\xi c_1 + c_2)(T' - A') \\
& - V_{tb}^* V_{ts}[3(\xi c_7 + c_8)(YT' - ZA') + \frac{3}{2}(\xi c_9 + c_{10})(T' - A')]\},
\end{aligned} \tag{34}
$$

where

$$
\begin{aligned}
Y &= \frac{m_K^2}{(m_b - m_u)(m_s + m_u)}, \\
Z &= \frac{m_B^2}{(m_b + m_u)(m_s - m_u)}, \\
C' &= i f_\pi [f_{BK}^+(m_\pi^2)(m_B^2 - m_K^2) + f_{BK}^-(m_\pi^2) m_\pi^2], \\
T' &= i f_K [f_{B\pi}^+(m_K^2)(m_B^2 - m_\pi^2) + f_{B\pi}^-(m_K^2) m_K^2], \\
A' &= i f_B [f_{K\pi}^+(m_B^2)(m_K^2 - m_\pi^2) + f_{K\pi}^-(m_B^2) m_B^2]. \qquad (35)
\end{aligned}
$$

The amplitudes $\bar{U}$ and $\bar{V}$ have the same form as U and V, respectively, except that the CP-conjugate amplitudes contain the complex conjugate CKM matrix elements $V_{ub}V_{us}^*$ and $V_{tb}V_{ts}^*$. As expected the strong and dipole penguin operators do not contribute to U and V. The only difference between the amplitudes (U and V) and the CP-conjugate cmplitudes ($\bar{U}$ and $\bar{V}$) is the opposite sign of weak phase angle γ. We note that if electroweak penguin contributions are neglected, we would find that the relation of eq.(32) holds. However, we now obtain the following ratios

$$\frac{|U - \bar{U}e^{2i\gamma}|}{|U_{tree}|} = 166\%|\sin\gamma| \qquad (36)$$

and

$$\frac{|V - \bar{V}e^{2i\gamma}|}{|V_{tree}|} = 42\%|\sin\gamma|, \qquad (37)$$

where the amplitudes U_{tree} and V_{tree} denote the tree contribution of U and V , respectively. Here we have used $m_s = 175$ MeV, $f_K = 162$ MeV, $f_B = 200$ MeV, and the form factors calculated in Ref. [23]. We conclude that the assumption presented in Ref. [9] is invalid and so the suggested isospin analysis is unworkable.

IV. MEASUREMENT OF THE ANGLE β

In this section we study penguin effects on the determination of the CKM phase β. Many methods have been suggested involving B meson decay into charmed particles. We consider two of the most convenient experimentally.

(1) Using $B \to \Psi K_s$

The easiest way to measure β is to measure the parameter $Im\lambda$ in CP asymmetry of time evolution in $B^0 \to \psi K_S$ [5]. In this case,

$$Im\lambda = Im(\frac{q}{p}\frac{\bar{A}(\bar{B}^0 \to \psi K_S)}{A(B^0 \to \psi K_S)}) \ . \tag{38}$$

The tree amplitude is proportional to $V_{cb}V_{cs}^*$, and the dominant penguin contribution from the internal top quark is proportional to $V_{tb}V_{ts}^*$, and so the decay amplitude can be parametrized as

$$A(\bar{B}^0 \to \psi K_S) = V_{cb}V_{cs}^* T_{\psi K} + V_{tb}V_{ts}^* P_{\psi K} \ . \tag{39}$$

Using unitarity of the CKM matrix, we can rewrite the above as

$$A(\bar{B}^0 \to \psi K_S) = V_{cb}V_{cs}^*(T_{\psi K} - P_{\psi K}) - V_{ub}V_{us}^* P_{\psi K} \ . \tag{40}$$

The WC's involved indicate that $|T_{\psi K}|$ is much larger than $|P_{\psi K}|$. Also $|V_{cb}V_{cs}^*|$ is about 50 times larger than $|V_{ub}V_{us}^*|$ from experimental data. We can, then, safely negelect the contribution from the term proportional to $V_{ub}V_{us}^*$. To a very good approximation even if the penguin (strong and electroweak) effects are included, $Im\lambda = Im((q/p)(V_{cb}V_{cs}^*/V_{cb}^*V_{cs})) = -\sin(2\beta)$. β can be measured with an error less than a percent.

(2) Using $B \to D^+D^-$ decay

The same conclusion can not be drawn for method to determine β by measuring the parameter $Im\lambda$ in the process $\bar{B}^0 \to D^+D^-$. In this case, the decay amplitude can be parametrized as

$$\begin{aligned} A(\bar{B}^0 \to D^+D^-) &= V_{cb}V_{cd}^* T_{DD} + V_{tb}V_{td}^* P_{DD} \\ &= V_{cb}V_{cd}^*(T_{DD} - P_{DD}) - V_{ub}V_{ud}^* P_{DD} \ . \end{aligned} \tag{41}$$

In this case although the penguin amplitude P_{DD} resulting from the top loop is suppressed compared with the tree amplitude T_{DD}, the CKM elements involved are comparable for each term. The error caused by penguin effects are much larger. If we keep the pegnuin contribution, we find

$$Im\lambda = -\frac{\sin(2\beta) + 2R_{DD}\sin(3\beta)\cos\delta + R_{DD}^2\sin(4\beta)}{1 + R_{DD}^2 + 2R_{DD}\cos(2\beta+\delta)} \ , \tag{42}$$

where $R_{DD} = |P_{DD}/T_{DD}|$, and δ is the relative strong rescattering phase between the tree and penguin amplitudes.

In factorization approximation we can calculate the ratio R_{DD}. For the decay $\bar{B}^0 \to D^+D^-$, we find

$$\begin{aligned}\bar{A}(\bar{B}^0 \to D^+D^-) = \frac{G_F}{\sqrt{2}}\{&V_{cb}V_{cd}^*(\xi c_1 + c_2) \\ &- V_{tb}V_{td}^*[\xi c_3 + c_4 + 2(\xi c_5 + c_6 + \xi c_7 + c_8)X' + (\xi c_9 + c_{10})]\tilde{T}',\end{aligned} \tag{43}$$

where

$$\begin{aligned}X' &= \frac{m_D^2}{(m_b - m_c)(m_c + m_d)}, \\ \tilde{T}' &= if_D[f_{BD}^+(m_D^2)(m_B^2 - m_D^2) + f_{BD}^-(m_D^2)m_D^2].\end{aligned} \tag{44}$$

Using the effective coefficients c_i, the masses $m_D = 1.869$ GeV, $m_c = 1.3$ GeV, the decay constant $f_D = 162$ MeV, and the form factors calculated in Ref. [23], we obtain

$$R_{DD} = 0.09 \ . \tag{45}$$

Here we have neglected a small contribution due to a u-quark loop in penguin diagram (about 8% compared with a top-quark loop contribution) and annihilation effects. In Fig.3. we plot $\Delta\sin(2\beta) = Im\lambda + \sin(2\beta)$ as a function of β. For the strong rescattering angle δ, we use the quark level estimate $\delta = 12.4^0$ by including absorptive contribution in the WC's. The error for certain values of β can be quite large. For example, for $\beta = 45^0$, the error is above 16%. We also carried out calculations with the dipole penguin operator contribution. We again find their effects to be small.

V. MEASUREMENT OF THE PHASE γ

In this scetion we comment on some methods for measuring the CKM phase angle γ. Several different classes of method to measure γ have been proposed. One class involve

charged B decays into a charge kaon and a neutral charmed meson [27]. This class of methods is not contaminated by penguin effects because only tree amplitude can mediate such decays. These should work well and we do not consider them here. Another class of method is to use information from $B \to \pi\pi$, and $B \to \pi(\eta)K$. In this case both tree and penguin amplitudes contribute, and care must be taken to include pegnuin effects.

A method to measure γ using these decays was first studied by Gronau, London and Rosner (GLR) [10]. In this method the tree and strong penguin effects were considered, but the electroweak penguin effects were neglected. GLR argued from isospin analysis that the combined amplitude $\bar{A}(B^- \to \pi^- \bar{K}^0) + \bar{A}(B^- \to \pi^0 K^-)$ is a pure $I = 3/2$ amplitude, and therefore only the tree amplitude contributes. Using SU(3) relation shown in eq.(10), this amplitude is found to be equal to the decay amplitude $(f_K/f_\pi)(V_{us}^*/V_{ud})^2 \bar{A}(B^- \to \pi^- \pi^0)$, which can be measured. These three amplitudes form a closed triangle, and similarly for their CP-conjugate amplitudes. Using the fact that $|\bar{A}(B^- \to \pi^- \bar{K}^0)| = |A(B^+ \to \pi^+ K^0)|$ (because the tree contributions here are negnigiblely small), from the two triangles for the particle and anti-particle decay amplitudes, the relative phase 2γ between $A(B^- \to \pi^- \pi^0)$ and $A(B^+ \to \pi^+ \pi^0)$ can be obtained. This is a very interesting proposal. However it was soon pointed out by Deshpande and He [12] that the inclusion of electroweak penguin effects invalidate this method because the electroweak contributions to $I = 3/2$ amplitude are compareable to the tree contribution.

Other methods have been proposed to take into account the electroweak penguin effects. Recently Gronau, Hernandez, London and Rosner (GHLR) [13] showed that the difficulty with the electroweak penguin effects can be solved by constructing the quadrilaterals discussed in section III for $B \to \pi K$ decays. As is already shown, the way used to construct the quadrilaterals in Ref. [9] is not workable. Instead, GHLR used SU(3) relation to relate one of the diagonal of the quadrilateral to $B_s^- \to K^- \eta$. This time the common side of the two quadrilaterals is chosen to be $|A(B^+ \to \pi^+ K^0)| = |\bar{A}(B^- \to \pi^- \bar{K}^0)|$. This method is however very difficult to implement experimentally because the decay amplitude for $B_s^- \to K^- \eta$ is dominated by electroweak penguin contribution and has a very small branching ratio [28].

A more practical method has recently been proposed by Deshpande and He [14] using SU(3) relations between the decay amplitudes for $\Delta S = 1$ decays $B^- \to \pi^- \bar{K}^0$, $\pi^0 K^-$, ηK^-, and $\Delta S = 0$ decay $B^- \to \pi^- \pi^0$. This method requires the construction of the triangles obtained from eq.(11)

$$\begin{aligned}
&\sqrt{2}\bar{A}(B^- \to \pi^0 K^-) - 2\bar{A}(B^- \to \pi^- \bar{K}^0) = \sqrt{6}\bar{A}(B^- \to \eta_8 K^-), \\
&\sqrt{2}A(B^+ \to \pi^0 K^+) - 2A(B^+ \to \pi^+ K^0) = \sqrt{6}A(B^+ \to \eta_8 K^+),
\end{aligned} \tag{46}$$

where η_8 is the pure octet component. Using the relation

$$\bar{A}(B^- \to \pi^- \bar{K}^0) = A(B^+ \to \pi^+ K^0), \tag{47}$$

the following result was obtained in Ref. [14]:

$$B - \bar{B} = -i2\sqrt{2}e^{i\delta^T}\frac{|V_{us}|}{|V_{ud}|}|\bar{A}(B^- \to \pi^- \pi^0)| \sin\gamma, \tag{48}$$

where B and $\bar{B}$ are the complex quantities defined as

$$\begin{aligned}
B &= \sqrt{2}\bar{A}(B^- \to \pi^0 K^-) - \bar{A}(B^- \to \pi^- \bar{K}^0), \\
\bar{B} &= \sqrt{2}A(B^+ \to \pi^0 K^+) - A(B^+ \to \pi^+ \bar{K}^0),
\end{aligned} \tag{49}$$

shown in Fig.4. The angle δ^T denotes the strong final state rescattering phase of the tree amplitude of B (or $\bar{B}$). Thus sinγ can be determined from eq.(48). This method is free from the electroweak penguin contamination problem, and all decays involved have relatively large ($O(10^{-5})$) branching ratios. They are within the reach of future experiments [29].

The results given in Ref. [14] hold in the exact SU(3) limit. This relation may be broken by SU(3) breaking effects due to η - η' mixing, the breaking effects in form factors and mass differences. We now make quantitative estimates of the influence of SU(3) breaking effects in the factorization approximation. In this approximation we find

$$\begin{aligned}
F_1 &\equiv \sqrt{2}A(B^- \to \pi^0 K^-) - 2A(B^- \to \pi^- \bar{K}^0) \\
&= \frac{G_F}{\sqrt{2}}\{V_{ub}V_{us}^*[(c_1 + \xi c_2)C' + (\xi c_1 + c_2)(T' - A')]
\end{aligned}$$

$$
\begin{aligned}
&- V_{tb}V_{ts}^*[-(\xi c_3 + c_4)(T' + A') - 2(\xi c_5 + c_6)(YT' + ZA') \\
&+ 2(\xi c_7 + c_8)(2YT' - ZA') - \frac{3}{2}(c_7 + \xi c_8 - c_9 - \xi c_{10})C' \\
&+ (\xi c_9 + c_{10})(2T' - A')]\},
\end{aligned}
$$

$$
\begin{aligned}
F_2 \equiv & \sqrt{6}A(B^- \to \eta_8 K^-) \\
= & \frac{G_F}{\sqrt{2}}\{V_{ub}V_{us}^*[(c_1 + \xi c_2)C'' + (\xi c_1 + c_2)(T'' - A'')] \\
& - V_{tb}V_{ts}^*[(\xi c_3 + c_4)(T'' - 2C'' - A'') + 2(\xi c_5 + c_6)(-2X''C'' + YT'' - ZA'') \\
& + 2(\xi c_7 + c_8)(X''C'' + YT'' - ZA'') - \frac{3}{2}(c_7 + \xi c_8 - c_9 - \xi c_{10})C'' \\
& + (\xi c_9 + c_{10})(C'' + T'' - A'')]\},
\end{aligned}
\tag{50}
$$

where C', T', A', Y and Z are given in eq.(35) and

$$
\begin{aligned}
X'' &= \frac{m_{\eta_8}^2}{2m_s(m_b - m_s)}, \\
C'' &= if_{\eta_8}[f_{BK}^+(m_{\eta_8}^2)(m_B^2 - m_K^2) + f_{BK}^-(m_{\eta_8}^2)m_{\eta_8}^2], \\
T'' &= if_K[f_{B\eta_8}^+(m_K^2)(m_B^2 - m_{\eta_8}^2) + f_{B\eta_8}^-(m_K^2)m_K^2], \\
A'' &= if_B[f_{K\eta_8}^+(m_B^2)(m_K^2 - m_{\eta_8}^2) + f_{K\eta_8}^-(m_B^2)m_B^2].
\end{aligned}
\tag{51}
$$

We note that in SU(3) limit the triangle relation eq.(46) is verified. In numerical estimates, neglecting small contribution of the annihilation diagram, we obtain

$$
\begin{aligned}
F_1 &= \frac{G_F}{\sqrt{2}}i[V_{ub}V_{us}^*(1.86GeV^3)e^{i\delta_T} + V_{tb}V_{ts}^*(-4.82 \times 10^{-2}GeV^3)e^{i\delta_P}] \quad , \\
F_2 &= \frac{G_F}{\sqrt{2}}i[V_{ub}V_{us}^*(2.70GeV^3)e^{i\delta_T} + V_{tb}V_{ts}^*(-4.79 \times 10^{-2}GeV^3)e^{i\delta_P}] \quad .
\end{aligned}
\tag{52}
$$

In the above we have inserted arbitrary strong rescattering phase in the amplitudes. We can estimate the phases using absorptive part in WC's which indicate small phase for δ_P and zero phase for δ_T. We however keep them as free parameters here for covenience. For our numerical values we have used the decay constants $f_{\eta_8} = 176$ MeV, $f_{\eta_0} \approx f_{\eta_8}$ (η_0 is the singlet component), and the from factors obtained in Ref. [23]. We see that the SU(3) breaking effects in the tree amplitude are about 30%, and much smaller effects in the penguin amplitudes.

To have an idea how large the SU(3) breaking effects on the determination of γ are, we carried out an excercise by taking $F_{1,2}$ in eq.(52) to be the experimental values keeping γ and $\delta = \delta_P - \delta_T$ as free parameters. For given values of γ and δ, we construct a triangle and obtain the value for $|B - \bar{B}|$. We then take the calculated amplitude for $|\bar{A}(B^- \to \pi^- \pi^0)|$ as the measured value, and use eq.(48) to determine $|\sin\gamma|$. For each given γ, using eq.(48) we will obtain a output γ. We will call it γ'. Corresponding to the cases $\gamma > \delta$ and $\gamma < \delta$, there are two solutions for $B - \bar{B}$ which arise from the two possible orientations of the two triangles relative to their common side. Fig. 4. shows the two triangles used to find the magnitude $|B - \bar{B}|$ for $\gamma > \delta$. We expect that because of the SU(3) breaking effects the triangle relation will have some deviation. This deviation will cause errors in determining $|\sin\gamma|$ and the angle γ. In Fig. 5. we show the errors $\Delta\gamma = \gamma - \gamma'$ for a fixed strong phase $\delta = 12^0$ which we expect from quark level evaluation of absorptive parts. Since δ is expected to be small we focus on the case $\gamma > \delta$. From Fig.5 we see that there are limited range where there is solution for γ' because a triangle can not be formed after breaking effects for all given γ and δ. We see that the errors $\Delta\gamma$ increases as γ increases. For instance, $\Delta\gamma/\gamma$ is within about 20% for $23^0 < \gamma < 35^0$. For larger values of γ the error is larger. If the form factors are varied by taking a different fit, it is possible to reduce errors. We conclude that this method is very sensitive to SU(3) breaking effects. More theoretical efforts to study SU(3) breaking effects are called for. We hope Lattice calculation will provide us with useful information on the evaluation of amplitudes.

VI. CONCLUSIONS

We have studied the influence of penguin (especially, electroweak penguin) contributions for several methods to extract the angles α, β, and γ of the CKM unitary triangle. Our calculations are based on the factorization approximation using the general effective Hamiltonian to the next-to-leading order and on some models for form factors when we have needed to obtain numerical values. To see the sensitivity of the results on the form factors

used, we repeated our calculations using form factors obtained in Ref. [30]. The conclusions are not changed for all cases except the calculation in Sec.V. Here the SU(3) breaking effects become larger.

In the cases of measuring the phase α using isospin relations for $B \to \pi\pi$ [7] and similarly for the method using time dependent asymmetries in $B \to \rho\pi$ [8], even though the electroweak penguin contributions contaminate the results, the phase α can be determined with an accuracy better than a few percent because the electroweak penguin effects are found to be quite small. However, for the approach proposed in Ref. [11], we have shown that the method is unworkable since in this case the electroweak penguin effects are comparable to the strong penguin effects. Similarly for method proposed in Ref. [9] to extract α from analysis of $B \to \pi K$, we found that the electroweak penguin contributions can not be neglected and the assumption of the analysis is again invalid.

For the measurement of β, the penguin effects are negligible when use $B \to \psi K_S$. We found that penguin effects are not so small in extracting $\sin(2\beta)$ from measurement of a CP-asymmetry in $B \to D^+D^-$. The deviation from $\sin(2\beta)$ due to penguin contributions will be over 10% if the value of β is in the range of $12^0 < \beta < 62^0$.

Finally we have made quantitative estimates of SU(3) breaking effects for the method of measuring the angle γ proposed in Ref. [14]. We found that the results are very sensitive to SU(3) breaking effects and permit extraction of γ only in a limited range of parameter space. More study of the SU(3) breaking effects on the decay amplitudes are called for. Recently, methods using nonet symmetry [15] have been suggested. These methods are subject to nonet symmetry breaking, and detailed studies are required to test their feasibility.

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FIGURES

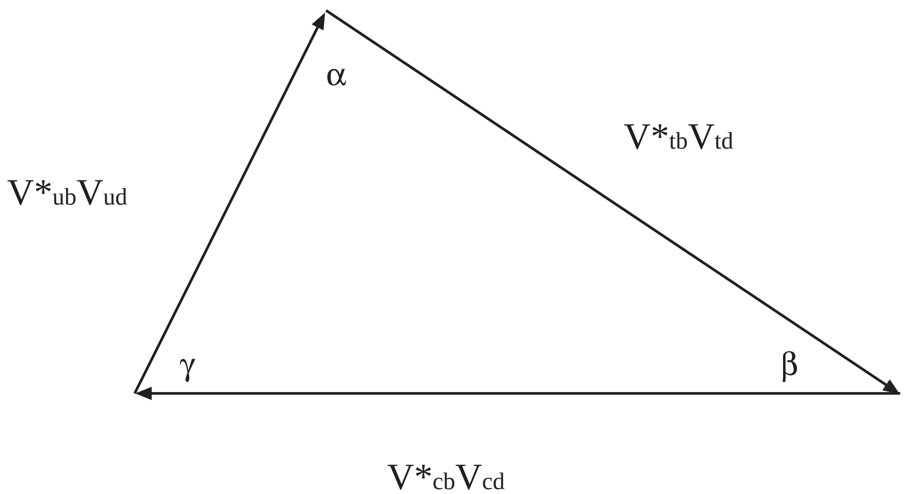

FIG. 1. The CKM unitarity triangle.

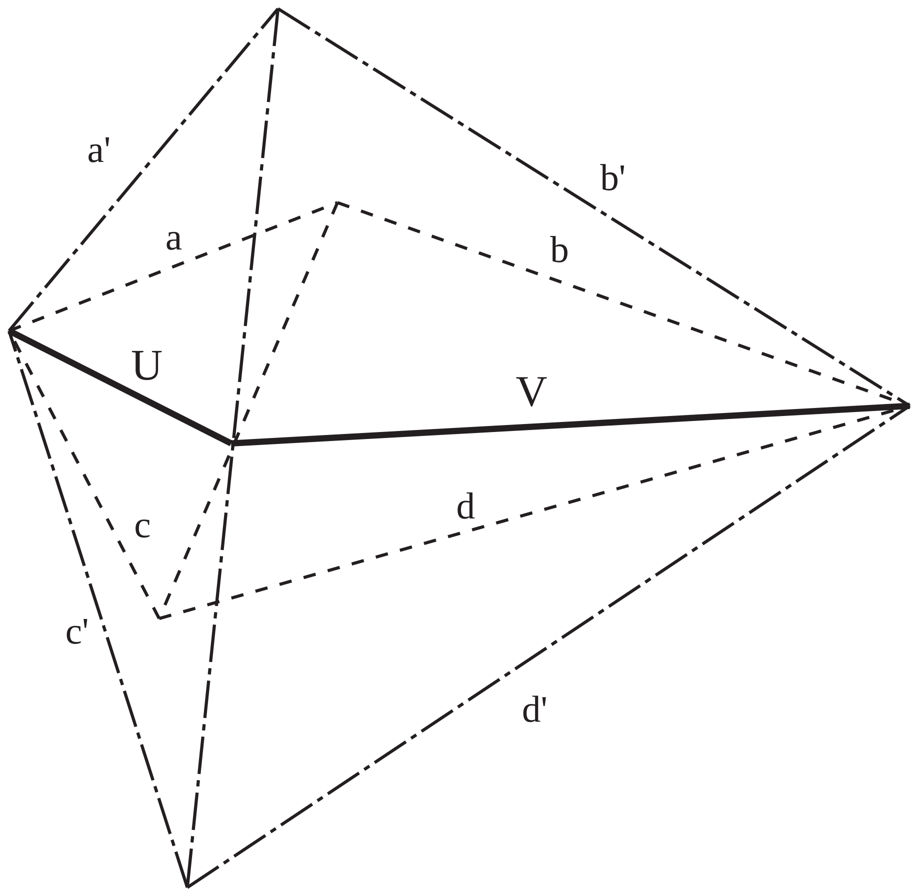

FIG. 2. The two quadrilaterals with a common diagonal $U+V$. Lines a, b, c, and d denote the amplitudes $A(B^0 \to \pi^0 K^0)$, $A(B^0 \to \pi^- K^+)$, $A(B^+ \to \pi^0 K^+)$, and $A(B^+ \to \pi^+ K^0)$. Similarly lines a', b', c', and d' denote the corresponding amplitudes $\tilde{A}$'s.

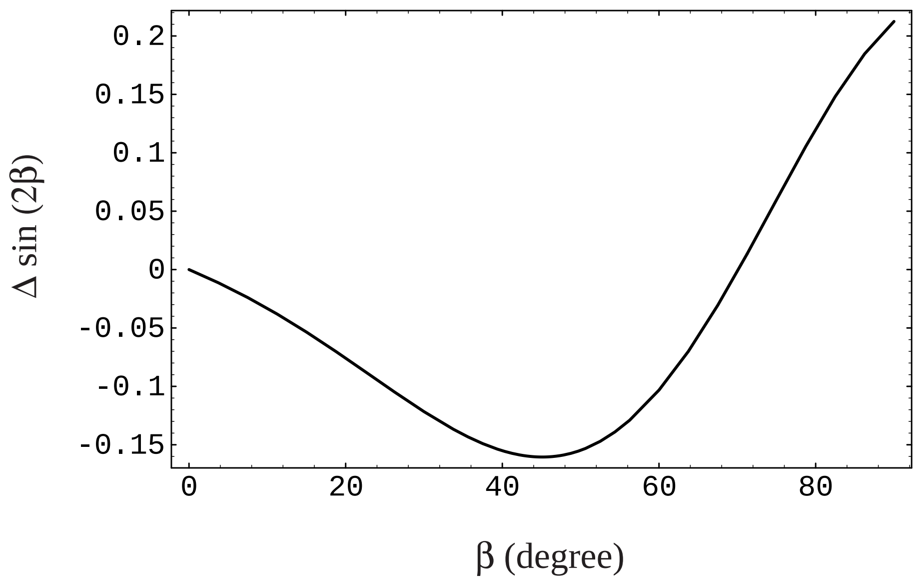

FIG. 3. $\Delta \sin(2\beta) = Im\lambda + \sin(2\beta)$ versus β for a fixed strong rescattering phase $\delta = 12.4^0$.

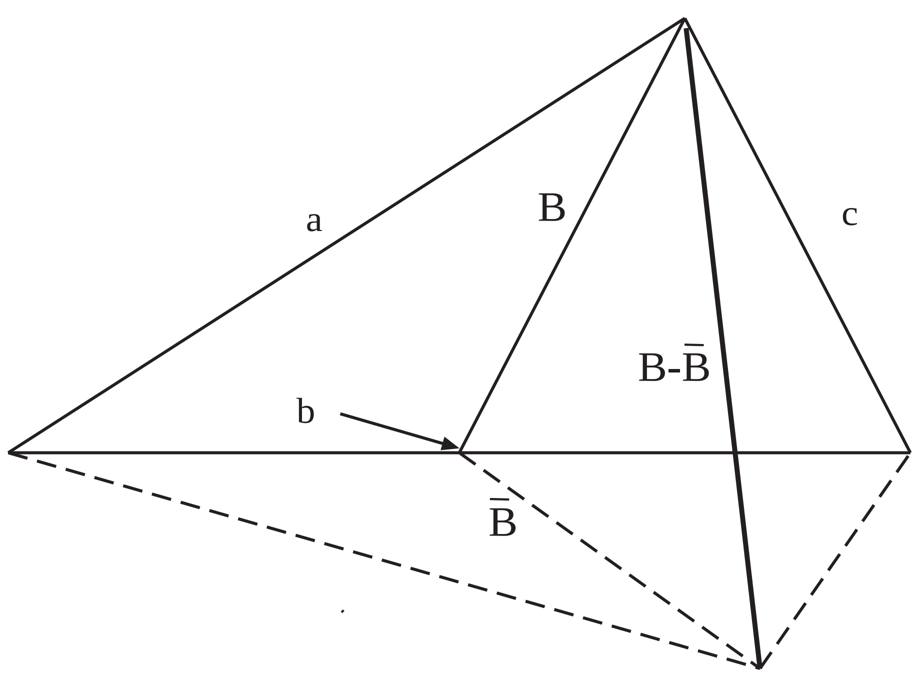

FIG. 4. The triangles used to find $|B - \bar{B}|$ for $\gamma > \delta$. Lines a, b, and c denote the amplitudes $2^{1/2}\bar{A}(B^- \to \pi^0 K^-)$, $\bar{A}(B^- \to \pi^- \bar{K}^0)$, and $6^{1/2}\bar{A}(B^- \to \eta_8 K^-)$. The dashed lines are for the corresponding B^+ decay amplitudes.

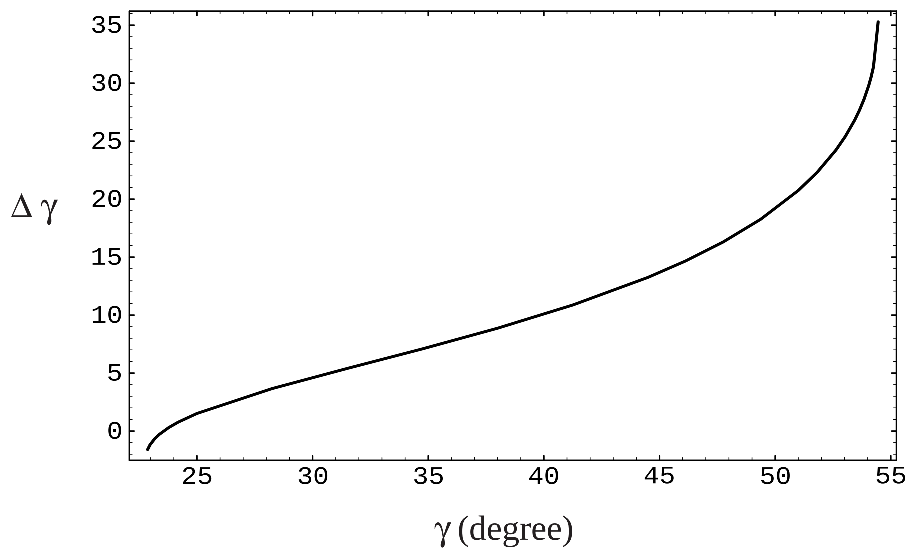

FIG. 5. $\Delta\gamma = \gamma - \gamma'$ versus γ for a fixed strong rescattering phase $\delta = 12^0$.